TE Wave Properties of Slab Dielectric Guide Bounded by Nonlinear Non-Kerr-Like Media

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***Abstract*— TE wave properties of slab dielectric waveguides with nonlinear non-Kerr-like substrate and cladding are presented. The guide can support even- and odd-symmetric modes in asymmetrical substrate and asymmetrical modes in completely symmetrical substrate. Dispersion relations and electric field profiles are illustrated and analytically discussed in detail.**

I. Introduction

It has been known since the early 1980s [3]–[6] that properties of waves guided by thin films take on striking new properties when one or more of the bounding media exhibits an intensity-dependent refractive index. With appropriate material conditions, both wave-vector and field distributions become strongly power dependent. Because a number of potential applications for such nonlinear waveguides to all optical signal processing have been identified in recent years with the development of technology, increasing attention has been devoted to these effects with a view to realizing these optical devices. These developments, in turn, have recently stimulated more realistic theoretical investigations of properties of nonlinear guided waves [1], [7]–[13]. With some exceptions (see e.g., [1], [5], [9], and [10]), many theoretical studies of nonlinear guided waves have been limited to Kerr-like nonlinear media ([3], [4], [6], [7], and [13] and references therein). Moreover, useful solutions of the nonlinear wave equation which include a field-dependent dielectric constant have been obtained for the Kerr-law case.

However, in real world cases, many materials exhibit a refractive index which varies with the electric field raised to a power other than two [1], [5]. The actual dependence of the index on the optical field is intimately related to the physical process which gives rise to nonlinearities, such as semiconductors diffusion and recombination effects, etc. In retrospect, the formalism necessary for generalizing the analysis of nonlinear slab-guided wave phenomena to non-Kerr-like media has been available for some time [1], [5], [9], and [10] and many numerical methods, such as beam propagation method [11], finite-element method [12], the first integration method [5], the phase-plane approach [1], and Runge–Kutta [2] have been employed.

Here, we systemetrically examine the variation in TE wave solutions with guided wave power, because the use of TE solutions has the strong theoretical advantage that Maxwell's equations reduce to a single fundamental nonlinear equation that, under an assumption of power-dependence with a form $\epsilon_r \sim |E|^\delta$, where δ is a arbitrary positive quantity, can be solved analytically. The structure discussed in this paper is a linear thin dielectric slab guide, with refractive index n_f, sandwiched between two nonlinear semi-infinite dielectrics. For the case of Kerr-like nonlinearity Boardman *et al.* [6] and Seaton *et al.* [3] have studied this structure numerically and analytically in detail, respectively. For the case of non-Kerr-like nonlinearity, which is the more practical case, this structure has been numerically discussed by Stegeman *et al.* [5] using the first integration method.

Although numerical methods are generally effective, they present also the disadvantage of not allowing physical interpretation of solutions. In nonlinear problems, in fact, the existence and identification of invariant quantities related to *observable* physical parameters of the configuration is particularly important. Using purely numerical methods, however, a qualitative analysis of the structure is not possible and many of its underlying features can not be perceived so clearly.

In this paper, a slab guide with a nonlinear cladding and substrate both having nonquadratic power-law dependent refractive index is analytically studied, and closed-form solutions will be given for the first time. They indicate that there are even- and odd-symmetric modes in this structure not only for symmetric cladding and substrate, but also for asymmetric cladding and substrate, and for completely symmetric cladding and substrate there are asymmetric modes too. Propagation properties of the guide are illustrated and discussed.

This paper is organized as follows: In the next section analytical solutions for special non-Kerr-like nonlinearity which varies with the electric field raised to a power other than two are given. The third section deals in detail with analytical results for even- and odd-symmetrical modes supported by this structure. General TE modes are presented in Section IV. Some general properties and conclusions are discussed in Section V. Since it is not feasible to examine all possibilities in detail, we will concentrate on those which are of interest for experiments and potential device applications.

II. Theoretical Approach and Analytical Solutions

The geometry considered in this paper is shown in Fig. 1. It is assumed that the film region ($-d < x < d$) has a linear refractive index n_f (related relative dielectric constant is ϵ_f) that is independent of the wave power. Both of the cladding and substrating media can exhibit a field-dependence relative dielectric constant of the form $\epsilon_c = n_c^2 + \alpha_c|E_c|^{\delta_c}$ or

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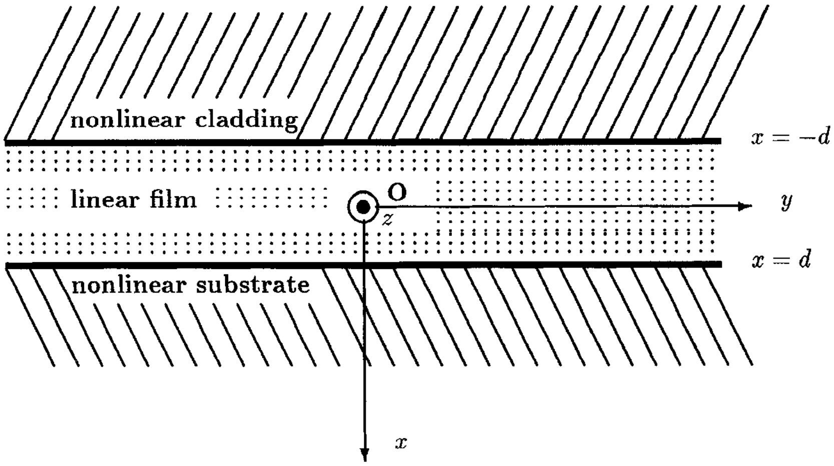

Fig. 1. The nonlinear guided wave geometry for the analysis.

$\epsilon_s = n_s^2 + \alpha_s |E_s|^{\delta_s}$ for the cladding and substrating medium, respectively. Wave fields investigated here are TE polarized having the form [5]

$$E(r,t) = \mathrm{Re}\{E(x)e^{i(\omega t - \beta z)}\}. \tag{1}$$

Only the component E_y of electric field is not zero in this case.

Using c, f, s referring to cladding, film and substrate, respectively, the field must satisfy the following equations [5]:

$$\frac{d^2E_{yc}}{dx^2} + (k_o^2 n_c^2 - \beta^2 + k_o^2 \alpha_c |E_{yc}|^{\delta_c})E_{yc} = 0, \quad (x \le -d) \tag{2}$$

$$\frac{d^2E_{yf}}{dx^2} + (k_o^2 n_f^2 - \beta^2)E_{yf} = 0, \qquad (-d \le x \le d) \tag{3}$$

$$\frac{d^2E_{ys}}{dx^2} + (k_o^2 n_s^2 - \beta^2 + k_o^2 \alpha_s |E_{ys}|^{\delta_s})E_{ys} = 0, \quad (d \le x) \tag{4}$$

where $k_o^2 = \omega^2 \mu_o \epsilon_o, \alpha_i \ (i = c, s)$ is the nonlinear coefficient. $\alpha_i > 0 \ (i = c, s)$ is named a focusing medium, and $\alpha_i < 0 \ (i = c, s)$ a defocusing medium.

Using normalized parameters recommended by Rozzi *et al.* [1]

$$k_c^2 = N^2 - n_c^2, \quad k_f^2 = n_f^2 - N^2, \quad k_s^2 = N^2 - n_s^2, \tag{5}$$

$$X = k_o x, \quad N = \frac{\beta}{k_o}, \quad a = k_o d \tag{6}$$

and

$$u_c(X) = |\alpha_c|^{1/\delta_c} E_{yc}(X), \quad u_f(X) = E_{yf}(X),$$
$$u_s(X) = |\alpha_s|^{1/\delta_s} E_{ys}(X) \tag{7}$$

thus the wave equations (2)–(4) become

$$\ddot{u}_c(X) - (k_c^2 - \sigma_c |u_c|^{\delta_c}) u_c(X) = 0, \qquad X \le -a \tag{8}$$

$$\ddot{u}_f(X) + k_f^2 u_f(X) = 0, \qquad -a \le X \le a \tag{9}$$

$$\ddot{u}_s(X) - (k_s^2 - \sigma_s |u_s|^{\delta_s}) u_s(X) = 0, \qquad a \le X \tag{10}$$

where for focusing media $\sigma_i = 1$ and for defocusing media $\sigma_i = -1 \ (i = c, s)$. $\ddot{(\ )} = d^2(\ )/dX^2$.

The analytical solution of $u_c(X)$ and $u_s(X)$ for focusing and defocusing media, respectively, is formed as follows:

For focusing medium ($\sigma_i = 1$), let

$$u_c(X) = \pm \frac{A_c}{\cosh^{\lambda_c}[B_c(X_c - a - X)]} \tag{11}$$

thus

$$\dot{u}_c = \lambda_c B_c \tanh[B_c(X_c - a - X)] u_c(X), \tag{12}$$

$$\ddot{u}_c = (\lambda_c B_c \tanh[B_c(X_c - a - X)])^2 u_c(X) - \lambda_c B_c^2 \frac{u_c(X)}{\cosh^2[B_c(X_c - a - X)]} \tag{13}$$

and

$$u_s(X) = \pm \frac{A_s}{\cosh^{\lambda_s}[B_s(X_s - a + X)]}, \tag{14}$$

$$\dot{u}_s = -\lambda_s B_s \tanh[B_s(X_s - a + X)] u_s(X), \tag{15}$$

$$\ddot{u}_s = (\lambda_s B_s \tanh[B_s(X_s - a + X)])^2 u_s(X) - \lambda_s B_s^2 \frac{u_s(X)}{\cosh^2[B_s(X_s - a + X)]} \tag{16}$$

where $A_i > 0 \ (i = c, s)$. Substituting (11)–(13) into (8)

$$(\lambda_c B_c \tanh[B_c(X_c - a - X)])^2 - \frac{\lambda_c B_c^2}{\cosh^2[B_c(X_c - a - X)]} - k_c^2 + \frac{A_c^{\delta_c}}{\cosh^{\delta_c \lambda_c}[B_c(X_c - a - X)]} = 0. \tag{17}$$

If (17) is satisfied, it must be

$$\lambda_c = \frac{2}{\delta_c}, \quad B_c = \frac{k_c}{\lambda_c} = \frac{\delta_c}{2} k_c,$$
$$A_c = \left(k_c^2 \frac{2 + \delta_c}{2}\right)^{1/\delta_c}. \tag{18}$$

Therefore, the solution of $u_c(X)$ in the cladding is

$$u_c(X) = \pm \frac{\left(k_c^2 \frac{2 + \delta_c}{2}\right)^{1/\delta_c}}{\left\{\cosh^2\left[\frac{\delta_c}{2} k_c (X_c - a - X)\right]\right\}^{1/\delta_c}}. \tag{19}$$

Similarly, the solution of $u_s(X)$ in the substrate is

$$u_s(X) = \pm\frac{\left(k_s^2\frac{2+\delta_s}{2}\right)^{1/\delta_s}}{\left\{\cosh^2\left[\frac{\delta_s}{2}k_s(X_s-a+X)\right]\right\}^{1/\delta_s}}. \tag{20}$$

For defocusing media ($\sigma_i = -1$), analytical solutions can be also obtained

$$u_c(X) = \pm\frac{\left(k_c^2\frac{2+\delta_c}{2}\right)^{1/\delta_c}}{\left\{\sinh^2\left[\frac{\delta_c}{2}k_c(X_c-a-X)\right]\right\}^{1/\delta_c}}, \tag{21}$$

$$u_s(X) = \pm\frac{\left(k_s^2\frac{2+\delta_s}{2}\right)^{1/\delta_s}}{\left\{\sinh^2\left[\frac{\delta_s}{2}k_s(X_s-a+X)\right]\right\}^{1/\delta_s}}. \tag{22}$$

Thus, solutions for non-Kerr-like nonlinear focusing and defocusing media are derived, respectively. Here X_c and X_s are parameters related to initial conditions. In (19) and (20) X_i $(i = c, s)$ can be positive, negative and zero. But for solutions of defocusing media X_c and X_s must be positive, because the function $1/\sinh(w)$ has a pole at $w = 0$. If $\delta_i = 2$ $(i = c, s)$ (Kerr-law dependence) solutions (19)–(22) are identical to those given by [3] and [4]. Using the derived analytical solutions electromagnetic wave propagation of the waveguide shown in Fig. 1 is studied.

III. Symmetric Modes

For symmetric modes (even modes) there is $E_f(-a) = E_f(a)$, and for anti-symmetric modes (odd modes) there is $E_f(-a) = -E_f(a)$.

A. Even Modes or Symmetric Modes

The solution of guided-waves in a linear film is well known

$$u_f(X) = A_f\cos(k_f X), \quad N < n_f. \tag{23}$$

Using boundary conditions at $X = \pm a$, for focusing medium we have

$$\tan(k_f a) = \frac{k_c}{k_f}\tanh\left(\frac{\delta_c X_c k_c}{2}\right), \tag{24}$$

$$= \frac{k_s}{k_f}\tanh\left(\frac{\delta_s X_s k_s}{2}\right), \tag{25}$$

$$A_f = \frac{1}{\alpha_c^{1/\delta_c}}\frac{u_c(-a)}{\cos(k_f a)}, \tag{26}$$

$$= \frac{1}{\alpha_s^{1/\delta_s}}\frac{u_s(a)}{\cos(k_f a)}. \tag{27}$$

It must be

$$k_c\tanh\left(\frac{\delta_c X_c k_c}{2}\right) = k_s\tanh\left(\frac{\delta_s X_s k_s}{2}\right), \quad \text{and}$$

$$\frac{u_c(-a)}{\alpha_c^{1/\delta_c}} = \frac{u_s(a)}{\alpha_s^{1/\delta_s}}. \tag{28}$$

For $\sigma_i < 0, \coth(\delta_c X_c k_c/2)$ and $\coth(\delta_s X_s k_s/2)$ replace the $\tanh(\delta_c X_c k_c/2)$ and $\tanh(\delta_s X_s k_s/2)$ functions in the above equations. The dispersion relation is:

$$a = k_o\, d = \frac{1}{k_f}\left(m\pi + \arctan\left(\frac{k_c'}{k_f}\right)\right), \quad m = (0), 1, 2, \cdots \tag{29}$$

with

$$k_c' = k_c\tanh\left(\frac{\delta_c X_c k_c}{2}\right), \quad \text{focusing-medium} \tag{30}$$

or

$$k_c' = k_c\coth\left(\frac{\delta_c X_c k_c}{2}\right). \quad \text{defocusing-medium.} \tag{31}$$

If $X_c \to \infty$, (29) becomes the linear dispersion equation. X_c can be both positive and negative, if $X_c \le 0, m$ must be larger than zero. Now the mode with $m = 0$ does not exist, and the mode with $m = 1$ becomes the fundamental mode.

In Fig. 2 dispersion relations of even symmetric modes are given. It shows if $X_c = 0$, curves for various δ_c are overlapping, and the $m = 0$ mode in nonlinear cases only exists for $X_c > 0$. If $X_c > 0$, the larger δ_c is, the smaller the difference between linear cases and nonlinear cases is. But for $X_c < 0$, the larger δ_c is, the larger the difference between linear cases and nonlinear cases is. If the cladding and the substrate are both focusing media, all curves of the nonlinear case are above those of the linear case, but for defocusing media curves of the linear case are above those of the nonlinear case as indicated in Fig. 2(c).

As an example, the electric field distributions of the even-mode TE_1 are shown in Fig. 3, the parameters are: $N = 1.560\,099, \delta_c = 1.0, 1.5, 2.0, 2.5, X_c = 5$ or $X_c = -5$ and $\sigma_i = 1$. It is shown that for $X_c = -5$ there is a maximum in the cladding and the substrate.

B. Odd-Modes or Antisymmetric Modes

In the case of odd modes the solution of guided-wave in the linear film is

$$u_f(X) = B_f\sin(k_f X), \quad N < n_f \tag{32}$$

and the dispersion relation is

$$a = k_o d = \frac{1}{k_f}\left[\left(m - \frac{1}{2}\right)\pi + \arctan\left(\frac{k_c'}{k_f}\right)\right] \tag{33}$$

also (28) must be satisfied.

In Fig. 4 dispersion curves of the odd TE guided modes are given for a focusing cladding and substrate.

C. Remarks

For a focusing media if X_c is zero, k_c' defined in (29) is zero, too. Now both dispersion relations of the even-modes (29) and the odd-modes (33) are dependent only on k_f. That means, they are independent of k_c, k_s and δ_c, δ_s. For even- and odd-symmetric modes, only the conditions given in (28) are needed. If $n_c = n_f, \delta_c = \delta_s, \alpha_c = \alpha_s, X_c = X_s$, these conditions are automatically satisfied. In the following the

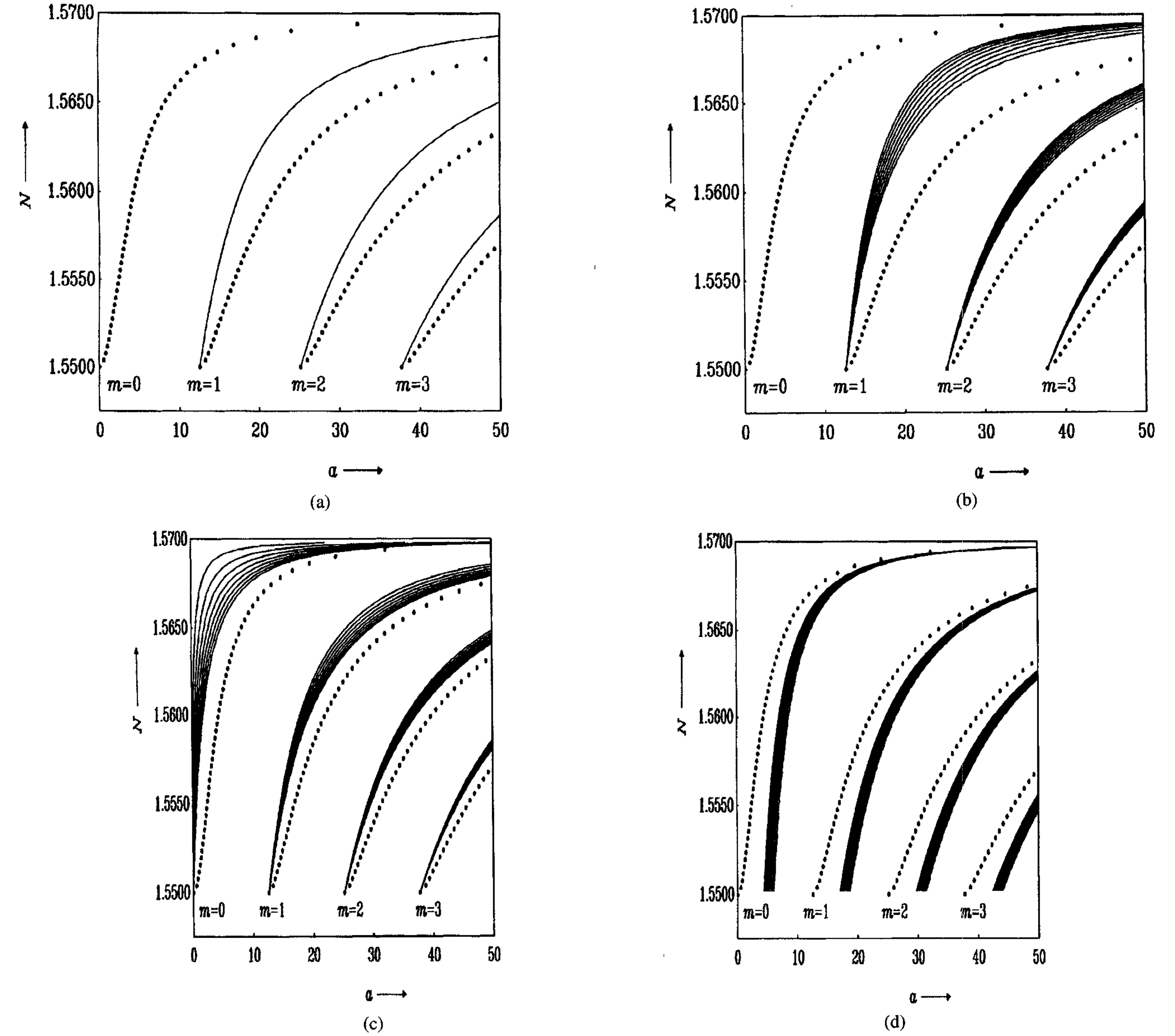

Fig. 2. Dispersion curves of even-modes for various X_c. $n_c = n_s = 1.55, n_f = 1.57$. Dotted lines: linear cases. δ_c is 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, and 4.0 from left to right for (c) and from right to left for (b) and (d), respectively, for each mode m. (a) $X_c = 0, \sigma = 1.$, (b) $X_c = -1, \sigma = -1.$, (c) $X_c = 1, \sigma = 1.$, (d) $X_c = 1, \sigma = -1$.

asymmetric case shall be studied. For simplicity only the case $\delta_c = \delta_s = \delta$ is discussed. Conditions in (28) become

$\sigma_c = \sigma_s = 1$:

$$k_c \tanh\left(\frac{\delta X_c k_c}{2}\right) = k_s \tanh\left(\frac{\delta X_s k_s}{2}\right) \quad (34)$$

and

$$\frac{1}{\sqrt{\alpha_c}} \frac{k_c}{\cosh\left(\frac{\delta X_s k_s}{2}\right)} = \frac{1}{\sqrt{\alpha_s}} \frac{k_s}{\cosh\left(\frac{\delta X_s k_s}{2}\right)} \quad (35)$$

where the solutions (19) and (20) have been used. And

$$\frac{\sinh(\delta X_s k_s/2)}{\sinh(\delta X_c k_c/2)} = \sqrt{\frac{\alpha_c}{\alpha_s}}. \quad (36)$$

If $X_c = 0$, it follows from (34) that X_s must be also zero; (35) is

$$\frac{k_c}{k_s} = \sqrt{\frac{\alpha_c}{\alpha_s}} \quad (37)$$

that is, if $n_c \neq n_s$, we can choose a suitable parameter pair α_c, X_c and α_s, X_s to satisfy (36), (37), or (28). Now the cladding and the substrate are not symmetric, but it can support even- and odd-symmetric modes. For given cladding and substrate, only these modes which have the normalized propagation constant N determined by (36) can exist in this structure with symmetric or anti-symmetric field distribution. As an example, in Fig. 5 the dependence $\alpha = \sqrt{\alpha_c/\alpha_s}$ versus X_s are drawn for $X_c = 10., n_c = 1.55, n_s = 1.54, n_f = 1.57, N = 1.560\,099, \sigma_c = \sigma_s = 1$ and $\delta = 0.5$, 1.0, 1.5, 2.0, 2.5, and 3.0. All of values in these curves satisfy (28) and

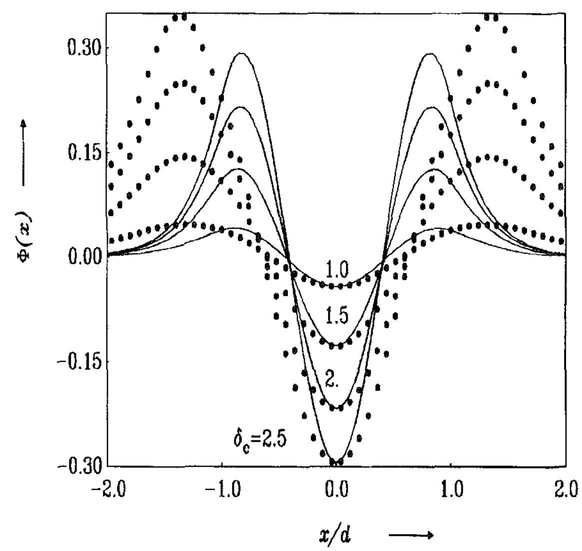

Fig. 3. Electric field distributions $\phi(x) = {}^{\delta}\!\sqrt{\alpha_c}\ E(x)$ of the TE_1-mode for focusing media. Solid lines: $X_c = X_s = 5$; Dotted lines: $X_c = X_s = -5, \delta_c = 1$, 1.5, 2, and 2.5.

can support both even- and odd-modes which have the same values N and $a = k_o d$.

Therefore, for completely asymmetric substrates even- and odd-modes can also propagate in nonlinear guides unlike in linear cases! For other cases $\sigma_c \neq \sigma_s$ and $\delta_c \neq \delta_s$ similar results can be also obtained.

From (29) and (33) it is known that the dispersion relation of the even- or odd-modes is only an explicit function of k'_c and it is not directly dependent on the parameter k'_s. k'_s can be determined from (36).

IV. General TE-Modes

A general solution for the field in the linear guide for guided waves is:

$$u_f(X) = A_f \cos(k_f X) + B_f \sin(k_f X). \tag{38}$$

Using solutions given in (19)–(22) and boundary conditions at $X = \pm a$ we have

$$A_f[k_f \sin(k_f a) - k'_c \cos(k_f a)] + B_f[k_f \cos(k_f a) + k'_c \sin(k_f a) = 0, \tag{39}$$

$$-A_f[k_f \sin(k_f a) - k'_s \cos(k_f a)] + B_f[k_f \cos(k_f a) + k'_s \sin(k_f a) = 0 \tag{40}$$

with

$$k'_i = k_i \tanh\left(\frac{\delta_i X_i k_i}{2}\right) \quad \text{for focusing-medium}, \quad (i = c, s), \tag{41}$$

$$k'_i = k_i \coth\left(\frac{\delta_i X_i k_i}{2}\right) \quad \text{for de-focusing-medium}, \quad (i = c, s). \tag{42}$$

If $k'_c = k'_s$, because (39) and (40) must be simultaneously satisfied, this leads to

$$A_f = 0, \quad B_f \neq 0 \quad \text{or} \quad A_f \neq 0, \quad B_f = 0 \tag{43}$$

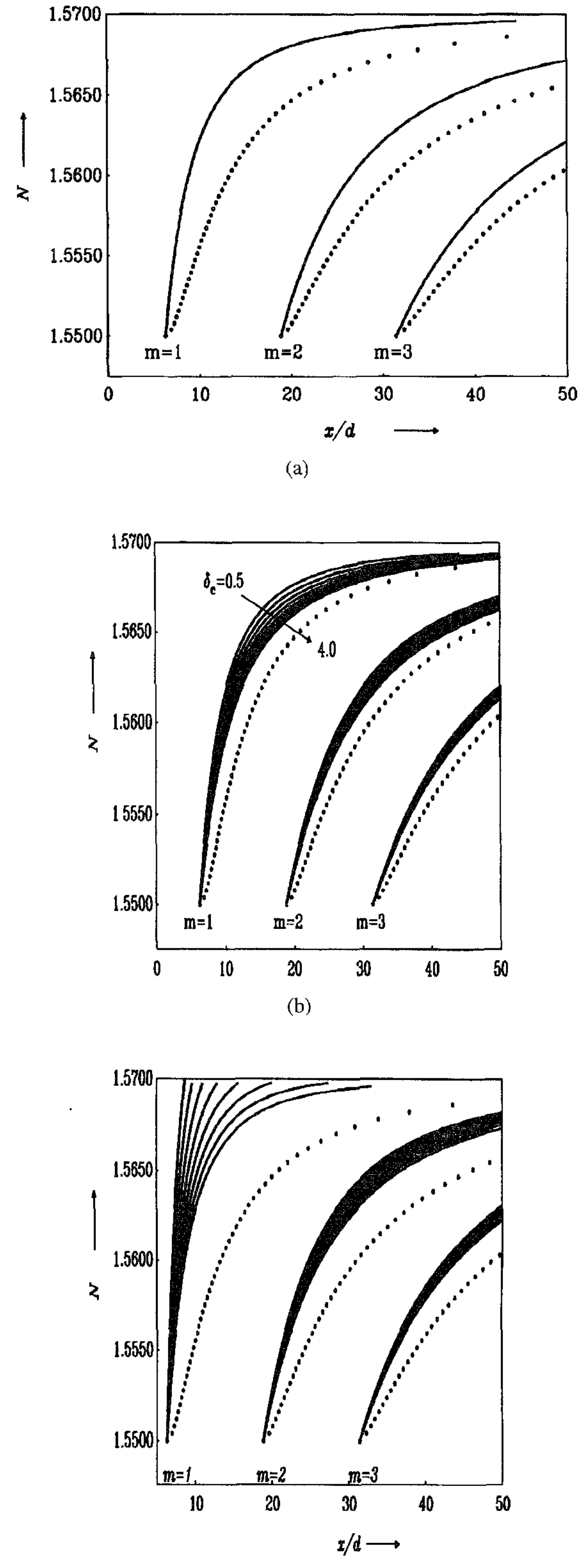

Fig. 4. Dispersion curves of odd TE modes. $\sigma_i = 1, n_c = n_s = 1.55$, $n_f = 1.57$. (a) $X_c = 0$. (b) $X_C = 1$. (c) $X_c = -1$.

in other words, these modes degenerate into odd-modes or even-modes as they have been discussed in the above section. That is, if and only if $k'_c = k'_s$, the structure can support even- and odd-modes.

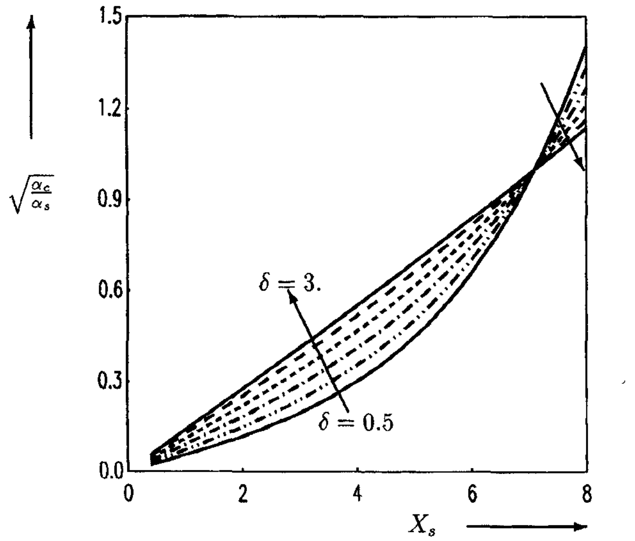

Fig. 5. $\sqrt{\alpha_c/\alpha_s}$ versus X_s for $X_c = 10, n_c = 1.55, n_s = 1.54, n_f = 1.57, N = 1.560099, \sigma_c = \sigma_s = 1$ and $\delta = 0.5$, 1.0, 1.5, 2.0, 2.5, and 3.0.

From (39) and (40) the dispersion equations can be derived

$$k_f a = m\pi + \arctan \cdot \left\{ \frac{\sqrt{[k_f^2 + (k_s')^2][k_f^2 + (k_c')^2]} - (k_f^2 - k_s' k_c')}{k_f(k_s' + k_c')} \right\}, \quad (m = (0), 1, 2 \cdots), \tag{44}$$

and

$$k_f a = m\pi - \arctan \cdot \left\{ \frac{\sqrt{[k_f^2 + (k_s')^2][k_f^2 + (k_c')^2]} + (k_f^2 - k_s' k_c')}{k_f(k_s' + k_c')} \right\}, \quad (m = (0), 1, 2, \cdots). \tag{45}$$

Here the existence of the mode with $m = 0$ depends on the parameters X_c and X_s. If $k_s' + k_c' > 0$, in (44) m can be zero, but in (45) m must >0. If $k_s' + k_c' < 0$, now in (44) m must >0, and in (45) m can be zero.

If $k_c' = k_s'$, (44) and (45) will be identical with (29) and (33), respectively.

Let $\Delta = k_c' - k_s' > 0$ and $(\Delta / k_c') \ll 1$, then (44) and (45) can approximately be written as

$$\tan(k_f a) \approx \frac{k_c'}{k_f} + \frac{\Delta}{k_f}, \tag{46}$$

and

$$\tan(k_f a) \approx -\frac{k_f}{k_c'}. \tag{47}$$

It shows that effects of Δ on even-modes are larger than those on odd-modes.

The derived dispersion equations (44) and (45) are valid for all possible combinations of the cladding and the substrate. Only k_c' and k_s' are different and they are given by (41) and (42).

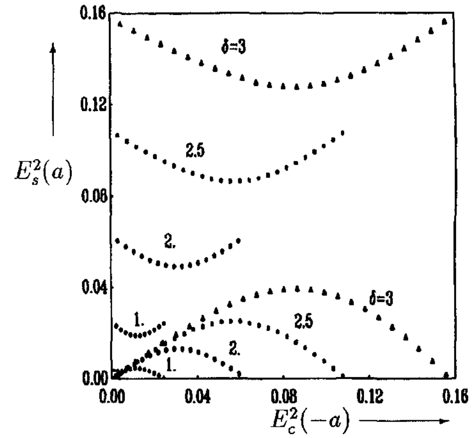

Fig. 6. Phase portrait of the asymmetric structure: $n_f = 1.57, n_c = 1.55, n_s = 1.54, \alpha_s/\alpha_c = 1.5, \delta = 3.0$, 2.5, 2.0, and 1.0, respectively.

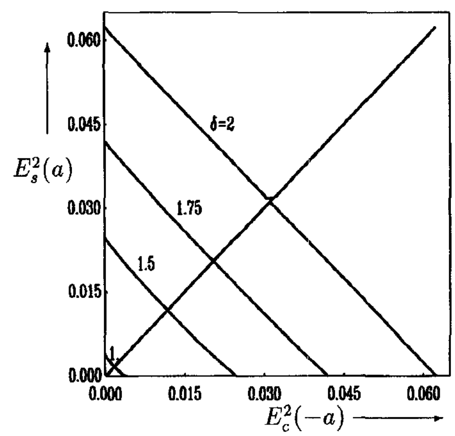

Fig. 7. Phase portrait of the completely symmetric structure: $n_f = 1.57, n_c = n_s = 1.55, \alpha_s/\alpha_c = 1., \delta = 1.0$, 1.5, 1.75, and 2.0.

If $k_c' \neq k_s'$, there will be no even- and odd-symmetric modes in this structure. In (41) and (42) the parameter X_i $(i = c, s)$ is dependent only on initial conditions and is neither a material nor a structure parameter. If the cladding and the substrate are really symmetric, that is, $\delta_c = \delta_s, n_c = n_s, \alpha_c = \alpha_s$, and $X_c \neq X_s$, we have $k_c' \neq k_s'$. That means only asymmetrical modes now can exist in the structure. In linear cases if and only if the cladding and the substrate are symmetric, even- and odd-modes can propagate in these guides. For Kerr-like cladding and substrate ($\delta_c = \delta_s = 2$) our conclusion consists with results given by Boardman *et al.* [6, pp. 1703] who have found that there are unusual asymmetric waves in a completely symmetric structure.

In Fig. 6 phase portraits for asymmetric structures are given. They indicate that for each set of determined structure parameters there are two branches. For the completely symmetric structure the phase portrait is shown in Fig. 7. It shows that there are either symmetric modes (diagonal line) or unusual asymmetric modes (other lines). Here $\delta_c = \delta_s$. From Fig. 7 it is seen that for a given value of $E_c^2(-a)$ there are two values of $E_s^2(a)$, one is for symmetric modes and the other for asymmetric modes.

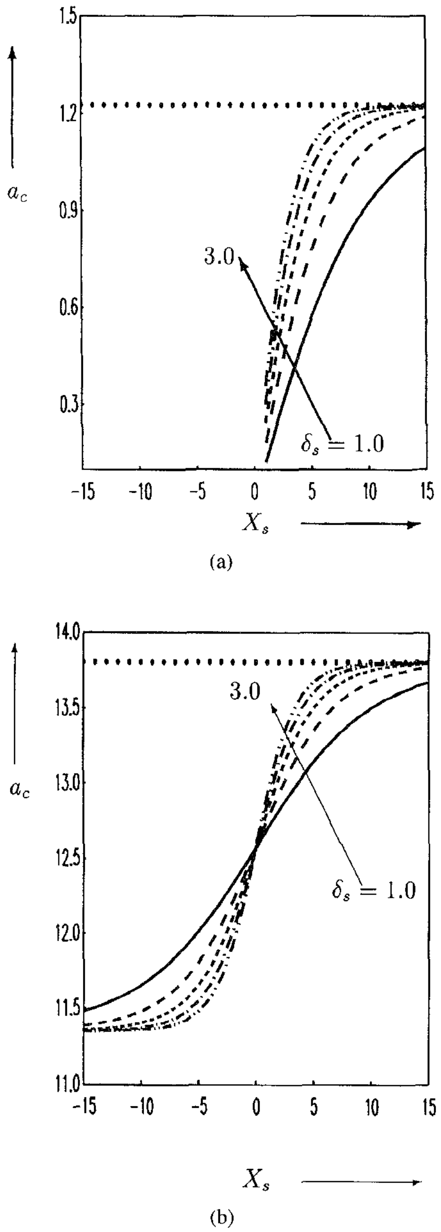

Fig. 8. The cutoff factor a_c versus X_s according to (44) with $n_f = 1.57, n_c = 1.55, n_s = 1.54, \delta_s = 1.0$, 1.5, 2.0, 2.5, and 3.0, respectively, for focusing substrate and $\sigma_c = 1$. • • • •: linear modes. (a) $m = 0$. (b) $m = 1$.

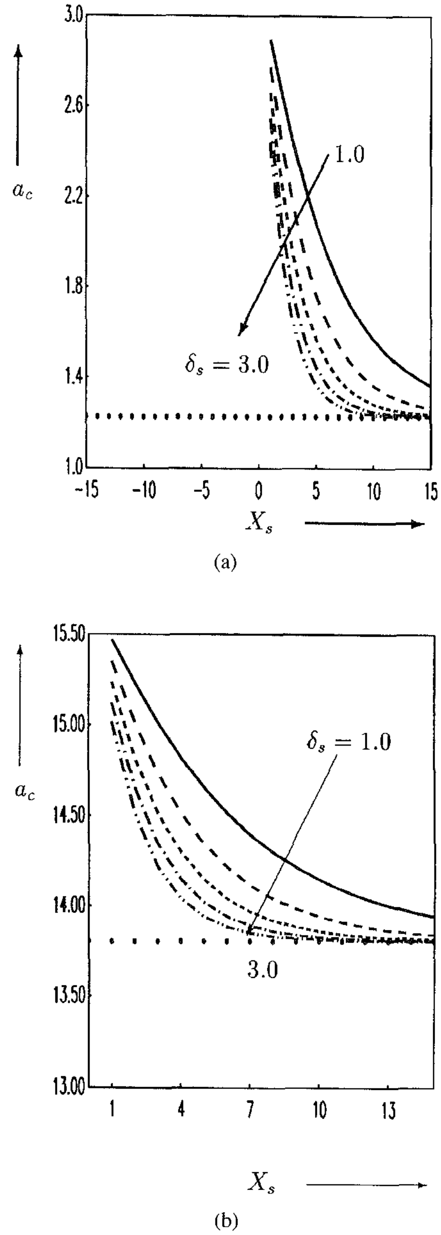

Fig. 9. a_c versus X_s for defocusing substrate. All parameters are the same as those in Fig. 5. (a) $m = 0$. (b) $m = 1$.

In following discussion, it is always assumed that $n_c > n_s$. Now the cut-off condition is $k_c = 0$ or $N = n_c$. From (44) and (45) the cut-off factor a_c can be simply obtained. For $\sigma_c = 1, k_c = 0 \Rightarrow k_c' = 0$, for $\sigma_c = -1, k_c = 0 \Rightarrow k_c' = (2/\delta_c X_c), (X_c > 0)$. Thus, for focusing cladding the cut-off factor a_c is independent of the parameters X_c, δ_c, but for defocusing cladding it is also a function of parameters X_c, δ_c. As examples, only cutoff conditions of modes $m = 0$ and $m = 1$ according to (44) with $k_c = 0$ are shown. Here $n_f = 1.57, n_c = 1.55, n_s = 1.54, \delta_s = 1.0$, 1.5, 2.0, 2.5, and 3.0 and the cladding is focusing medium. For focusing substrate the dependencies a_c versus X_s are given in Fig. 8(a) and (b) for $m = 0$ and $m = 1$, respectively, and for defocusing substrate a_c versus X_s is given in Fig. 9. For modes with $m > 1$ it can straightforwardly be obtained with $a_{c,m} = a_{c,1} + m\pi/\sqrt{n_f^2 - n_c^2}$.

If $X_s \to \infty$ or $\delta_s \to \infty$, values of a_c will approach those in the linear cases. According to (45) similar figures can also be easily obtained.

For linear cases dispersion curves of guided waves in the $N \sim a$ plane are a set of discrete curves, but with nonlinear cases, because there are many parameters for each given mode m, dispersion curves can vary in a region, named the allowed region. In Fig. 10 allowed and forbidden regions according to (44) are given for $n_c = 1.55, n_s = 1.54, n_f = 1.57$. In the regions below the solid lines, guided waves have no maximum in the cladding or substrate. In the regions above the solid lines guided waves have a maximum in the cladding or the substrate or in both the cladding and the substrate. All dispersion curves

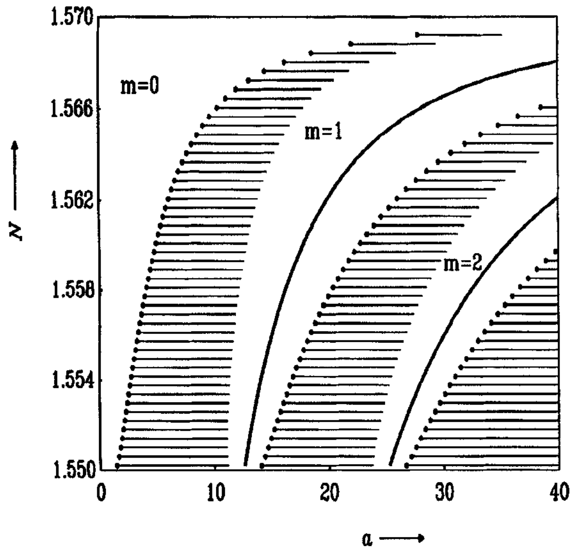

Fig. 10. Allowed and forbidden regimes according to (44). Shaded region: forbidden regions; —: $X_c = X_s = 0$; • • • •: linear curves. From the left to right, $m = 0, m = 1, m = 2$.

with possible δ_c, δ_s, X_c and X_s are located in allowed regions. According to (45) similar allowed and forbidden regimes can also be easily obtained.

The guided wave power in Watts per meter along the z-axis is given in terms of the Poynting vector in the usual way by [5]

$$P = \frac{\beta}{2}\sqrt{\frac{\epsilon_o}{\mu_o}}\int_{-\infty}^{\infty} E_y^2(x)\,dx \tag{48}$$

or

$$P = \frac{N}{2}\sqrt{\frac{\epsilon_o}{\mu_o}}\int_{-\infty}^{\infty} E_y^2(X)\,dX = P_c + P_f + P_s. \tag{49}$$

Using the electric field solutions we have

$$P_f = \frac{N}{2}\sqrt{\frac{\epsilon_o}{\mu_o}}a\left\{A_f^2 + B_f^2 + \frac{\sin(2k_f a)}{2k_f a}(A_f^2 - B_f^2)\right\} \tag{50}$$

with

$$A_f = \frac{E_{ys}(a) + E_{yc}(-a)}{2\cos(k_f a)}, \tag{51}$$

$$B_f = \frac{E_{ys}(a) - E_{yc}(-a)}{2\sin(k_f a)} \tag{52}$$

where $E_{ys}(X)$ and $E_{yc}(X)$ are defined in (19)–(22) and (7).

For $\sigma_i = 1$ $(i = c, s)$

$$P_i = \frac{N}{\delta_i k_i}\sqrt{\frac{\epsilon_o}{\mu_o}}\left(\frac{k_i^2}{\alpha_i}\frac{2+\delta_i}{2}\right)^{2/\delta_i}\int_{a1}^{\infty}\frac{dt}{\cosh^{4/\delta_i}(t)}, \quad i = c, s \tag{53}$$

where $a1 = ak_i(\delta_i/2)$. For $\sigma_i = -1$ $(i = c, s)$ the expression of P_i can be easily derived replacing $\sinh(t)$ by $\cosh(t)$ in

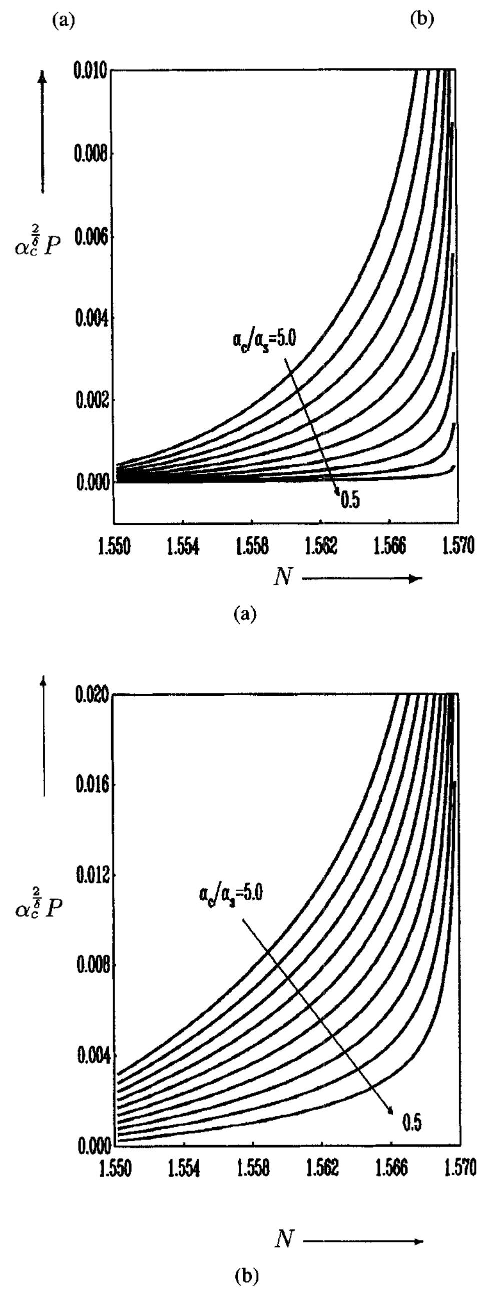

Fig. 11. $\alpha_c^{2/\delta}P$ versus N at $X_i = 0, \delta_c = \delta_s = \delta$. α_c/α_s is 5.0, 4.5, 4.0, 3.5, 3.0, and 2.5. (a) $\delta = 0.5$. (b) $\delta = 1$.

(53). (53) can be integrated analytically only for some values of δ_i:

$\sigma_i = 1$ $(i = c, s)$:

$$P_i = \sqrt{\frac{\epsilon_o}{\mu_o}}\frac{N}{k_i}2\left(\frac{1.25k_i^2}{\alpha_i}\right)^4\left(\frac{16}{35} - \tanh\frac{k_iX_i}{4} + \tanh^3\frac{k_iX_i}{4} - 0.6\tanh^5\frac{k_iX_i}{4} + \frac{1}{7}\tanh^7\frac{k_iX_i}{4}\right), \quad \text{for } \delta_i = 0.5, \tag{54}$$

$$P_i = \sqrt{\frac{\epsilon_o}{\mu_o}}Nk_i^3\left(\frac{1.5}{\alpha_i}\right)^2\left(\frac{2}{3} - \tanh\frac{k_iX_i}{2} + \frac{1}{3}\cdot\tanh^3\frac{k_iX_i}{2}\right), \quad \text{for } \delta_i = 1, \tag{55}$$

$$P_i = \sqrt{\frac{\epsilon_o}{\mu_o}} N \frac{k_i}{\alpha_i}(1 - \tanh(k_i X_i)), \quad \text{for } \delta_i = 2, \tag{56}$$

$$P_i = \sqrt{\frac{\epsilon_o}{\mu_o}} \frac{N}{2} \sqrt{\frac{3}{\alpha_i}} \left(\frac{\pi}{4} - \arctan[\tanh(k_i X_i)]\right), \quad \text{for } \delta_i = 4. \tag{57}$$

$\delta_i = 2$ is the Kerr-like medium and expression (56) are the same as equation (7) given by Seaton *et al.* in [3] who have studied the same structure with Kerr-like nonlinear cladding and substrate.

In Fig. 11 curves of $\alpha_c^{2/\delta} P$ versus N with $X_i = 0, \delta_c = \delta_s = \delta$ and for various α_c/α_s are given. Here $n_f = 1.57, n_c = 1.55, n_s = 1.54$ and $\sigma_i = 1$ $(i = c, s)$. From up to down α_c/α_s is 5.0, 4.5, 4.0, 3.5, 3.0, 2.5, 2.0, 1.5, 1.0, and 0.5.

For other parameters similar curves of power P versus N can be easily obtained using (53).

V. Conclusion

Guided waves in nonlinear planar waveguides have been studied intensively. Nonlinear media which exhibit a field-dependent refractive index which is proportional to the field raised to some arbitrary power were analytically investigated. It is obvious, from the derived analytic expressions that the design of nonlinear guided wave devices which rely on power-dependent changes in the field distribution depend strongly on the form of nonlinearities.

Results show that for the completely symmetric structure there are unusual asymmetric modes and for asymmetric structure there are symmetric mode solutions. If a correct field distribution with related initial parameters is launched in the absence of loss, the wave should maintain that field distribution as it propagates down the waveguide. Explicit analytical expressions and illustrations given here can be used for the design of the optical devices based on nonlinear waveguide structures.

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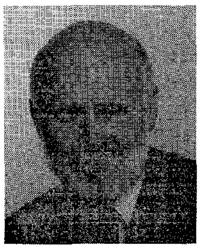

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Since 1992 he has lead the Institute of Mobile and Satellite Communication Techniques, Kamp-Lintfort, along with his university position. This institute intensively works in the area of mobile communication techniques, microwave and millimeter-wave techniques, antenna techniques, and electromagnetic compatibility techniques.